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DETERMINANT INEQUALITIES FOR HADAMARD PRODUCT OF POSITIVE DEFINITE MATRICES

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Abstract. Let A_i, $i=1,\ldots,m$, be $n\times n$ positive definite matrices whose diagonal blocks are n_j-square matrices $A_i^{(j)}$, $j=1,\ldots,k$. Choi recently proved

$$\det\left(\sum_{i=1}^{m}A_i^{-1}\right)\geqslant\det\left(\sum_{i=1}^{m}(A_i^{(1)})^{-1}\right)\cdots\det\left(\sum_{i=1}^{m}(A_i^{(k)})^{-1}\right).$$

We first give a new proof of this inequality, and then present an analogous inequality involving the Hadamard product

$$\det\left(\prod_{i=1}^{m}\circ A_i^{-1}\right)\geqslant\det\left(\prod_{i=1}^{m}\circ(A_i^{(1)})^{-1}\right)\cdots\det\left(\prod_{i=1}^{m}\circ(A_i^{(k)})^{-1}\right).$$

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