DETERMINANT INEQUALITIES FOR HADAMARD PRODUCT OF POSITIVE DEFINITE MATRICES

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Abstract. Let \( A_i, i = 1, \ldots, m \), be \( n \times n \) positive definite matrices whose diagonal blocks are \( n_j \)-square matrices \( A_i^{(j)} \), \( j = 1, \ldots, k \). Choi recently proved
\[
\det \left( \sum_{i=1}^{m} A_i^{-1} \right) \geq \det \left( \sum_{i=1}^{m} (A_i^{(1)})^{-1} \right) \cdots \det \left( \sum_{i=1}^{m} (A_i^{(k)})^{-1} \right).
\]
We first give a new proof of this inequality, and then present an analogous inequality involving the Hadamard product
\[
\det \left( \prod_{i=1}^{m} \circ A_i^{-1} \right) \geq \det \left( \prod_{i=1}^{m} \circ (A_i^{(1)})^{-1} \right) \cdots \det \left( \prod_{i=1}^{m} \circ (A_i^{(k)})^{-1} \right).
\]


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REFERENCES