

ILLUMINATED CAPS OF THE UNIT SPHERES OF BANACH SPACES AND RELATED INEQUALITIES

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Abstract. Geometric properties of illuminated caps of the unit sphere of a finite dimensional Banach space play an important role in Hadwiger's covering problem for centrally symmetric convex bodies. In view of this, fundamental properties of illuminated caps are presented, incenters, inradii, and self-circumradii of illuminated caps are studied, and related inequalities are obtained.

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