

A NEW SCHAUDER BASIS FOR $L^r((0, 1)^n)$, $n = 2, 3$

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Abstract. We show that under suitable conditions on p , q , and summability, the system of generalized trigonometric functions $\{\prod_{i=1}^d \sin_{p,q}(n_i \pi_{p,q} x_i)\}_{n_1, \dots, n_d}$ is a basis for $L^r((0, 1)^d)$ for any $r \in (1, \infty)$ where $d = 2$ or $d = 3$.

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