

VARIOUS INEQUALITIES RELATED TO THE ADAMS INEQUALITY ON WEIGHTED MORREY SPACES

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Abstract. We consider various inequalities related to the Adams inequality for the fractional integral operators on weighted Morrey spaces. In 2014, Izumi, Komori-Furuya and Sato proved an inequality which is the type of the Adams inequality on weighted Morrey spaces. Firstly, we investigate another proof of their result. Secondly, we investigate various inequalities related to their result for higher order commutators generated by BMO-functions and the fractional integral operator on weighted Morrey spaces. One of the main results in this paper recovers the result due to Cruz-Uribe and Fiorenza in 2003. Thirdly, we extend the fractional integrals to the multilinear fractional integrals. The result of the multilinear fractional integrals partially recovers the Moen result.

Mathematics subject classification (2010): 26A33, 42B25.

Keywords and phrases: Weights, weighted Morrey spaces, fractional integral operator, multilinear fractional integral operator, homogeneous kernels, BMO spaces, commutator.

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