

COMPLETELY MONOTONIC FUNCTIONS RELATED TO GURLAND'S RATIO FOR THE GAMMA FUNCTION

CHAO-PING CHEN AND JUNESANG CHOI

Abstract. The subject of complete monotonicity for functions has gained considerable popularity and importance from a rather long ago up to now, due mainly to its demonstrated applications in getting diverse inequalities. Here, we investigate some completely monotonic functions related to Gurland's ratio for the gamma function. Certain relevant connections of the results presented here with those earlier ones are also pointed out. Further, an interesting open conjecture regarding our present concern is posed.

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