

UL'YANOV-TYPE INEQUALITIES AND EMBEDDINGS BETWEEN BESOV SPACES: THE CASE OF PARAMETERS WITH LIMIT VALUES

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Abstract. In this paper we obtain some limit cases of inequalities of Ul'yanov-type for modulus of smoothness between Lorentz-Zygmund spaces on \mathbb{T}^n . Corresponding embedding theorems for the Besov spaces are investigated.

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