

WEIGHTED COMPOSITION OPERATOR ON TWO-DIMENSIONAL LORENTZ SPACES

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Abstract. The boundedness, compactness, closed range and invertibility of the weighted composition operator on two-dimensional Lorentz spaces are characterized.

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