SOME EXACT BERNSTEIN–SZEGŐ INEQUALITIES ON THE STANDARD TRIANGLE

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Abstract. An actual problem in the theory of approximations is to extend the univariate inequality of Bernstein to the multivariate setting. This question is satisfactorily settled in the case of a centrally symmetric convex body. In spite of the presence of good estimates, exact inequalities of Bernstein’s type for non-symmetric convex bodies are not known.

We prove that the approach based on the Krein-Milman theorem can be applied to maximize the nonlinear functional, which corresponds to the estimate of Bernstein-Szegő type for the gradients of arbitrary polynomials on convex bodies.

As applications we prove exact Bernstein-Szegő inequalities for some classes of bivariate polynomials on the standard triangle $\Delta$. Note that in a certain sense $\Delta$ is the least symmetric convex body in $\mathbb{R}^2$.

Keywords and phrases: Bernstein-Szegő inequalities, polynomials, extreme points.

REFERENCES