

## SOME EXACT BERNSTEIN–SZEGŐ INEQUALITIES ON THE STANDARD TRIANGLE

LOZKO MILEV AND NIKOLA NAIDENOV

*Abstract.* An actual problem in the theory of approximations is to extend the univariate inequality of Bernstein to the multivariate setting. This question is satisfactorily settled in the case of a centrally symmetric convex body. In spite of the presence of good estimates, exact inequalities of Bernstein’s type for nonsymmetric convex bodies are not known.

We prove that the approach based on the Krein–Milman theorem can be applied to maximize the nonlinear functional, which corresponds to the estimate of Bernstein–Szegő type for the gradients of arbitrary polynomials on convex bodies.

As applications we prove exact Bernstein–Szegő inequalities for some classes of bivariate polynomials on the standard triangle  $\Delta$ . Note that in a certain sense  $\Delta$  is the least symmetric convex body in  $\mathbb{R}^2$ .

*Mathematics subject classification* (2010): 41A17, 26C05, 26B25.

*Keywords and phrases:* Bernstein–Szegő inequalities, polynomials, extreme points.

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