BOUNDS FOR THE RADII OF UNIVALENCE
OF SOME SPECIAL FUNCTIONS

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Abstract. Tight lower and upper bounds for the radii of univalence (and starlikeness) of some normalized Bessel, Struve and Lommel functions of the first kind are obtained via Euler-Rayleigh inequalities. It is shown also that the radius of univalence of the Struve functions is greater than the corresponding radius of univalence of Bessel functions. Moreover, by using the idea of Kreyszig and Todd, and Wilf it is proved that the radii of univalence of some normalized Struve and Lommel functions are exactly the radii of starlikeness of the same functions, and they are actually solutions of some functional equations. The Laguerre-Pólya class of entire functions plays an important role in our study.

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