

## $L_p$ -MIXED AFFINE SURFACE AREA

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*Abstract.* Lutwak introduced the notion of  $L_p$ -affine surface area by  $L_p$ -mixed volume and obtained some related inequalities. In this article, based on the  $L_p$ -mixed quermassintegrals, we define the concept of the  $L_p$ -mixed affine surface area and extend some of Lutwak's result.

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