

SOME FAMILIES OF GENERALIZED MATHIEU–TYPE POWER SERIES, ASSOCIATED PROBABILITY DISTRIBUTIONS AND RELATED INEQUALITIES INVOLVING COMPLETE MONOTONICITY AND LOG–CONVEXITY

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Abstract. By making use of the familiar Mathieu series and its generalizations, the authors derive a number of new integral representations and present a systematic study of probability density functions and probability distributions associated with some generalizations of the Mathieu series. In particular, the mathematical expectation, variance and the characteristic functions, related to the probability density functions of the considered probability distributions are derived. As a consequence, some interesting inequalities involving complete monotonicity and log-convexity are derived.

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