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PROPERTIES OF THE TURÁNIAN OF MODIFIED BESSEL FUNCTIONS

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Abstract. In this paper some new series and integral representations for the Turánian of modified Bessel functions of the first kind are given, which give new asymptotic expansions and tight bounds for the Turán determinant in the question. It is shown that in the case of natural and real order the Turánian can be represented in a relatively compact form, which yields a uniform upper bound for the Turán determinant for modified Bessel functions of the first kind. Our results complement and improve some of the results from the literature.

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