

## NEW LOWER BOUNDS FOR ARITHMETIC, GEOMETRIC, HARMONIC MEAN INEQUALITIES AND ENTROPY UPPER BOUND

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*Abstract.* In this paper the arithmetic, geometric and harmonic mean inequalities are refined. New lower bounds for the corresponding inequalities which are better than the previous ones are obtained. As an application for the results, a strong upper bound for Shannon's entropy is presented. The new entropy upper bound improves the basic results of Simic (2009) and Țăpuș and Popescu (2012).

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