

ESTIMATES FOR TSALLIS RELATIVE OPERATOR ENTROPY

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Abstract. We give the tight bounds of Tsallis relative operator entropy by using Hermite-Hadamard's inequality. Some reverse inequalities related to Young's inequality are also given. In addition, operator inequalities for normalized positive linear map with Tsallis relative operator entropy are given.

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