A GRONWALL INEQUALITY FOR A
GENERAL CAPUTO FRACTIONAL OPERATOR

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Abstract. In this paper we present a new type of fractional operator, which is a generalization of the Caputo and Caputo–Hadamard fractional derivative operators. We study some properties of the operator, namely we prove that it is the inverse operation of a generalized fractional integral. A relation between this operator and a Riemann–Liouville type is established. We end with a fractional Gronwall inequality type, which is useful to compare solutions of fractional differential equations.

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