

## TWO-WEIGHT INEQUALITIES FOR GEOMETRIC MAXIMAL OPERATORS

ADAM OSĘKOWSKI

*Abstract.* We study one- and two-weight inequalities for the geometric maximal operator on probability spaces equipped with a tree-like structure. We provide a characterization of weights, in terms of Muckenhoupt and Sawyer-type conditions, for which the appropriate strong-type estimates hold. Our approach rests on Bellman function method, which allows us to identify sharp constants involved in the estimates.

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