INEQUALITIES FOR ZERO–BALANCED GAUSSIAN HYPERGEOMETRIC FUNCTIONS

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Abstract. In this paper, we consider the monotonicity of certain combinations of the Gaussian hypergeometric functions $F(a-1,b;a+b;1-x^c)$ and $F(a-1-\delta,b+\delta;a+b;1-x^d)$ on $(0,1)$ for $\delta \in (a-1,0)$, and study the problem of comparing these two functions, thus get the largest value $\delta_1 = \delta_1(a,c,d)$ such that the inequality $F(a-1,b;a+b;1-x^c) < F(a-1-\delta,b+\delta;a+b;1-x^d)$ holds for all $x \in (0,1)$.

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