PROOFS OF CERTAIN CONJECTURES OF VUKŠIĆ CONCERNING THE INEQUALITIES FOR MEANS

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Abstract. By using the asymptotic expansion method, Vukšić conjectured inequalities between Seiffert means and convex combinations of other means. In this paper, we prove certain conjectures given by Vukšić.

Mathematics subject classification (2010): 26E60, 26D05.
Keywords and phrases: Seiffert means, inequality.

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