NECESSARY CONDITIONS FOR THE BOUNDEDNESS OF LINEAR AND BILINEAR COMMUTATORS ON BANACH FUNCTION SPACES

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Abstract. In this article we extend recent results by the first author [3] on the necessity of *BMO* for the boundedness of commutators on the classical Lebesgue spaces. We generalize these results to a large class of Banach function spaces. We show that with modest assumptions on the underlying spaces and on the operator T, if the commutator $[b,T]$ is bounded, then the function b is in *BMO*.

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