Mathematical
Inequalities
& Applications
Volume 21, Number 1 (2018), 17–24

doi:10.7153/mia-2018-21-02

AN INFINITE SEQUENCE OF INEQUALITIES INVOLVING SPECIAL VALUES OF THE RIEMANN ZETA FUNCTION

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Abstract. In this paper, we give an infinite sequence of inequalities involving the Riemann zeta function with even arguments $\zeta(2n)$ and the Chebyshev-Stirling numbers of the first kind. This result is based on a recent connection between the Riemann zeta function and the complete homogeneous symmetric functions [18]. An interesting asymptotic formula related to the nth complete homogeneous symmetric function is conjectured in this context:

$$h_n\left(1,\left(\frac{k}{k+1}\right)^2,\left(\frac{k}{k+2}\right)^2,\ldots\right)\sim\binom{2k}{k},\qquad n\to\infty.$$

Mathematics subject classification (2010): 05E05, 11M06, 26D15.

Keywords and phrases: Inequalities, Chebyshev-Stirling number, Riemann zeta function, symmetric functions.

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