CONTINUITY AND APPROXIMATE DIFFERENTIABILITY OF MULTISUBLINEAR FRACTIONAL MAXIMAL FUNCTIONS

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Abstract. In this note we investigate the continuity and approximate differentiability of the $m$-sublinear fractional maximal operator

$$
\mathfrak{M}_\alpha(\tilde{f})(x) = \sup_{r>0} |B(x,r)|^{\alpha/d-m} \prod_{i=1}^m \int_{B(x,r)} |f_i(y)| \, dy,
$$

where $m \geq 1$, $0 \leq \alpha < md$ and $\tilde{f} = (f_1, \ldots, f_m)$ with each $f_j \in L^1_{\text{loc}}(\mathbb{R}^d)$. More precisely, we prove that $\mathfrak{M}_\alpha$ maps $W^{1,p_1}(\mathbb{R}^d) \times \cdots \times W^{1,p_m}(\mathbb{R}^d)$ into $W^{1,q}(\mathbb{R}^d)$ continuously, provided that $1 < p_1, \ldots, p_m < \infty$ and $0 < \sum_{i=1}^m 1/p_i - \alpha/d = 1/q$ $\leq 1$. We also show that the multisublinear fractional maximal functions $\mathfrak{M}_\alpha(\tilde{f})$ are approximately differentiable a.e. if $\tilde{f} = (f_1, f_2, \ldots, f_m)$ with each $f_j \in L^1(\mathbb{R}^d)$ being approximately differentiable a.e. As applications, the corresponding results for fractional maximal operators are established.


Keywords and phrases: Multisublinear fractional maximal operator, Sobolev space, continuity, approximate differentiability.

REFERENCES


