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MAXIMAL AND SINGULAR INTEGRAL OPERATORS AND THEIR COMMUTATORS ON GENERALIZED WEIGHTED MORREY SPACES WITH VARIABLE EXPONENT

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Abstract. We consider the generalized weighted Morrey spaces $\mathcal{M}_{\omega}^{p(\cdot),\varphi}(\Omega)$ with variable exponent $p(x)$ and a general function $\varphi(x,r)$ defining the Morrey-type norm. In case of unbounded sets $\Omega \subset \mathbb{R}^n$ we prove the boundedness of the Hardy-Littlewood maximal operator and Calderón-Zygmund singular operators with standard kernel, in such spaces. We also prove the boundedness of the commutators of maximal operator and Calderón-Zygmund singular operators in the generalized weighted Morrey spaces with variable exponent

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Paper MIA-21-04

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