

COVERING UNIT SPHERES AND BALLS OF NORMED SPACES BY SMALLER BALLS

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Abstract. We amend a widely used result given by Doyle, Lagarias, and Randall concerning the side length of equilateral Minkowski m -gons inscribed in the unit circle of a Minkowski plane. Based on this, we obtain the smallest positive number γ such that the unit circle S_X of a Minkowski plane X can be covered by m translates of γB_X , where B_X is the unit ball of X . Moreover, we improve a recent estimation of the smallest positive number γ such that the unit ball B_X of a Minkowski space X can be covered by m translates of γB_X .

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