UPPER AND LOWER BOUNDS, AND OPERATOR MONOTONICITY
OF AN EXTENSION OF THE PETZ–HASEGAWA FUNCTION

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Abstract. The Petz-Hasegawa function

$$f_p(x) = p(1-p)\frac{(x-1)^2}{(x^p-1)(x^{1-p}-1)}$$

for $p \in [-1,2]$ is a well-known operator monotone function on $x > 0$. In this paper, we discuss some properties of the following extension of the Petz-Hasegawa function

$$f_p(x) = x^\gamma \prod_{i=1}^{n} p_i \frac{x-1}{x^{p_i}-1},$$

where $p = (p_1,\ldots,p_n)$ by only using an elementary technique. Firstly, we get its upper and lower bounds. Secondly, we obtain a result on operator monotonicity.

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REFERENCES

[1] L. Cai and F. Hansen, *Metric-adjusted skew information: convexity and restricted forms of super-additivity*, Lett. Math. Phys. **93**, 1 (2010), 1–13.

[2] J. I. Fujii, *Interpolatinality for symmetric operator means*, Sci. Math. Jpn. **75**, 3 (2012), 267–274.

[3] J. I. Fujii and M. Fujii, *Upper estimations on integral operator means*, Sci. Math. Jpn. **75**, 2 (2012), 217–222.

[4] T. Furuta, *Elementary proof of Petz-Hasegawa theorem*, Lett. Math. Phys. **101**, 3 (2012), 355–359.

[5] F. Hiai and D. Petz, *Introduction to matrix analysis and applications*, Universitext. Springer, Cham; Hindustan Book Agency, New Delhi.

[6] S. Izumino and N. Nakamura, *Elementary proofs of operator monotonicity of some functions*, Sci. Math. Jpn. **77**, 3 (2015), 363–370.

[7] M. Nagisa and S. Wada, *Operator monotonicity of some functions*, Linear Algebra Appl. **486**, 1 (2015), 389–408.

[8] D. Petz and H. Hasegawa, *On the Riemannian metric of α-entropies of density matrices*, Lett. Math. Phys. **38**, 2 (1996), 221–225.

[9] Y. Udagawa, S. Wada, T. Yamazaki and M. Yanagida, *On a family of operator means involving the power difference means*, Linear Algebra Appl. **485**, 1 (2015), 124–131.