UPPER AND LOWER BOUNDS, AND OPERATOR MONOTONICITY OF AN EXTENSION OF THE PETZ–HASEGAWA FUNCTION

Takayuki Furuta, Masatoshi Ito, Takeaki Yamazaki and Masahiro Yanagida

Abstract. The Petz-Hasegawa function
\[ f_p(x) = p(1-p) \frac{(x-1)^2}{(x^p-1)(x^{1-p}-1)} \]
for \( p \in [-1, 2] \) is a well-known operator monotone function on \( x > 0 \). In this paper, we discuss some properties of the following extension of the Petz-Hasegawa function
\[ f_p(x) = x^\gamma \prod_{i=1}^{n} p_i \frac{x-1}{x^{p_i}-1}, \]
where \( p = (p_1, \ldots, p_n) \) by only using an elementary technique. Firstly, we get its upper and lower bounds. Secondly, we obtain a result on operator monotonicity.

Mathematics subject classification (2010): 47A63, 47A64.

Keywords and phrases: Positive definite operator, operator mean, operator monotone function, Petz-Hasegawa theorem.

REFERENCES