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RELATIONS BETWEEN THE GENERALIZED BESSEL FUNCTIONS AND THE JANOWSKI CLASS

S. KANAS, S. R. MONDAL AND A. D. MOHAMMED

Abstract. We are interested in finding the sufficient conditions on A, B, λ, b and c which ensure that the generalized Bessel functions $u_\lambda := u_{\lambda,b,c}$ satisfies the subordination $u_\lambda(z) \prec (1+Az)/(1+Bz)$. Also, conditions for which $u_\lambda(z)$ to be Janowski convex, and $zu'_\lambda(z)$ to be Janowski starlike in the unit disk $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ are obtained.

Mathematics subject classification (2010): 34B30, 33C10, 30C80, 30C45.

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