

HARDY TYPE INEQUALITIES AND COMPACTNESS OF A CLASS OF INTEGRAL OPERATORS WITH LOGARITHMIC SINGULARITIES

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Abstract. We establish criteria for both boundedness and compactness for some classes of integral operators with logarithmic singularities in weighted Lebesgue spaces for cases $1 < p \leq q < \infty$ and $1 < q < p < \infty$. As corollaries some corresponding new Hardy inequalities are pointed out.

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