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ON THE LEIBNIZ RULE FOR RANDOM VARIABLES

ZOLTÁN LÉKA

Abstract. We prove a Leibniz-type inequality for the spread of (real-valued) random variables in terms of their L^p-norms. The result is motivated by the Kato–Ponce inequality and Rieffel's Leibniz property.

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