

A COMPLEMENT TO DIANANDA'S INEQUALITY

PENG GAO

Abstract. Let $M_{n,r} = (\sum_{i=1}^n q_i x_i^r)^{\frac{1}{r}}$, $r \neq 0$ and $M_{n,0} = \lim_{r \rightarrow 0} M_{n,r}$ be the weighted power means of n non-negative numbers x_i with $q_i > 0$ satisfying $\sum_{i=1}^n q_i = 1$. In particular, $A_n = M_{n,1}$, $G_n = M_{n,0}$ are the arithmetic and geometric means of these numbers, respectively. A result of Diananda shows that

$$M_{n,1/2} - qA_n - (1-q)G_n \geq 0,$$

$$M_{n,1/2} - (1-q)A_n - qG_n \leq 0,$$

where $q = \min q_i$. In this paper, we prove analogue inequalities in the reversed direction.

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