

## ON THE HARMONIC AND GEOMETRIC MAXIMAL OPERATORS

LINDEN ANNE DUFFEE AND KABE MOEN

*Abstract.* We examine the harmonic and geometric maximal operators defined for a general basis of open sets in  $\mathbb{R}^n$ . We prove two weight norm inequalities for the harmonic maximal operator assuming testing conditions over characteristic functions of unions of sets from the basis. We also prove that a bumped two weight  $A_p$ -like condition is sufficient for the two weight boundedness of the harmonic maximal operator.

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