NORM INEQUALITIES AND CHARACTERIZATIONS
OF INNER PRODUCT SPACES

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Abstract. Let $(X, \|.\|)$ be a real normed space and let $\theta : (0, \infty) \to (0, \infty)$ be an increasing function such that $t \mapsto \frac{t}{\theta(t)}$ is non-decreasing on $(0, \infty)$. For such function, we introduce the notion of θ-angular distance $\alpha_\theta[x, y]$, where $x, y \in X \setminus \{0\}$, and show that X is an inner product space if and only if $\alpha_\theta[x, y] \leqslant 2 \frac{\|x-y\|}{\theta\|x\| + \theta\|y\|}$ for each $x, y \in X \setminus \{0\}$. Then, in order to generalize the Dunkl-Williams constant of X [10], we introduce a new geometric constant $C_F(X)$ for X wrt F, where $F : (0, \infty) \times (0, \infty) \to (0, \infty)$ is a given function, and obtain some characterizations of inner product spaces related to the constant $C_F(X)$. Our results generalize and extend various known results in the literature.

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REFERENCES

[1] A. M. AL-RASHED, *Norm inequalities and characterizations of inner product spaces*, J. Math. Anal. Appl., **176** (1993), 587–593.

[2] D. AMIR, *Characterizations of Inner Product Spaces*, Springer Basel, 1986.

[3] G. BIRKHOFF, *Orthogonality in linear metric spaces*, Duke Math., **1** (1935), 169–172.

[4] J. A. CLARKSON, *Uniformly convex spaces*, Trans. Amer. Math. Soc., **40** (1936), 396–414.

[5] F. DADIPOUR AND M. S. MOSLEHIAN, *A characterization of inner product spaces related to the p-angular distance*, J. Math. Anal. Appl., **371**, 2 (2010), 677–681.

[6] M. M. DAY, *Some characterizations of inner product spaces*, Trans. Amer. Math. Soc., **62** (1947), 320–337.

[7] H. DEHGHAN, *A characterization of inner product spaces related to the skew-angular distance*, Math. Notes., **93**, 4 (2013), 556–560.

[8] C. F. DUNKL AND K. S. WILLIAMS, *A simple norm inequality*, Amer. Math. Monthly., **71**, 1 (1964), 53–54.

[9] R. C. JAMES, *Inner products in normed linear spaces*, Bull. Amer. Math. Soc., **53** (1947), 559–566.

[10] A. JIMENEZ-MELADO, E. LLORENS-FUSTER AND E. M. MAZCUÑÁN-NAVARRO, *The Dunkl-Williams constant, convexity, smoothness and normal structure*, J. Math. Anal. Appl., **342**, 1 (2008), 298–310.

[11] P. JORDAN AND J. VON NEUMANN, *On inner products in linear metric spaces*, Ann. Math., **36**, 2 (1935), 719–723.

[12] W. A. KIRK AND M. F. SMILEY, *Another characterization of inner product spaces*, Amer. Math. Monthly., **71**, 8 (1964), 890–891.

[13] E. R. LORCH, *On certain implications which characterize Hilbert space*, Ann. Math., **49**, 3 (1948), 523–532.

[14] L. MALIGRANDA, *Simple norm inequalities*, Amer. Math. Monthly., **113**, 3 (2006), 256–260.

[15] H. MIZUGUCHI, K.-S. SAITO AND R. TANAKA, *On the calculation of the Dunkl-Williams constant of normed linear spaces*, Cent. Eur. J. Math., **11**, 7 (2013), 1212–1227.

[16] R. TANAKA, T. OHWADA AND K.-S. SAITO, *Geometric constants and characterizations of inner product spaces*, Math. Inequal. Appl., **17**, 2 (2014), 513–520.

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