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MORE ACCURATE CLASSES OF JENSEN–TYPE INEQUALITIES FOR CONVEX AND OPERATOR CONVEX FUNCTIONS

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Abstract. Motivated by a recent refinement of the scalar Jensen inequality obtained via linear interpolation, in this paper we develop a general method for improving two classes of Jensen-type inequalities for bounded self-adjoint operators. The first class refers to a usual convexity, while the second one deals with the operator convexity. The general results are then applied to quasi-arithmetic and power operator means. As a consequence, we obtain strengthened forms of the inequalities between arithmetic, geometric and harmonic operator means. We also obtain more accurate Young-type inequalities for unitarily invariant norms as well as more precise relations for some important jointly concave mappings.

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