

## POLYNOMIAL AND MULTILINEAR HARDY—LITTLEWOOD INEQUALITIES: ANALYTICAL AND NUMERICAL APPROACHES

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*Abstract.* We investigate the constants of the polynomial and multilinear Hardy–Littlewood inequalities. Among other results, we show that a simple application of the best known constants of the Clarkson inequality improves a recent result of Araújo et al. In a final section, as an independent appendix, we present some computer-aided estimates for the lower bounds of the multilinear Hardy–Littlewood inequalities.

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