

INEQUALITIES AND BOUNDS FOR A CERTAIN BIVARIATE ELLIPTIC MEAN II

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Abstract. Further investigations of the bivariate elliptic mean introduced in [?] are presented. In particular new bounds for the mean under discussion are obtained. Also, the Wilker and Huygens-type inequalities as well as the Landen transformation are established. Results presented in this work are complimentary to those derived recently in [?] and [?].

Mathematics subject classification (2010): 26E30, 26D05, 33C05.

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