Mathematical Inequalities & Applications
Volume 21, Number 2 (2018), 353–367 doi:10.7153/mia-2018-21-26

THE NORMALIZED L_p INTERSECTION BODIES

WEI WANG AND NAN ZHANG

Abstract. In this paper, we define the normalized L_p intersection body and prove that the normalized L_p intersection body operator is GL(n) contravariant of weight 0. We show that the polar body operator can be obtained as a limit of the normalized L_p intersection body operator. And we establish a dual Brunn-Minkowski type inequality for normalized L_p intersection bodies. Furthermore, the normalized L_p-Busemann-Petty problem is shown.

Mathematics subject classification (2010): 52A40, 53A15.
Keywords and phrases: Star body, radial function, L_p radial sum, L_p harmonic Blaschke radial sum, L_p dual mixed volumes.

REFERENCES

[1] G. BERCK, *Convexity of L_p-intersection bodies*, Adv. Math. **222** (2009), 920–936.
[2] S. CAMPI, *Convex intersection bodies in three and four dimensions*, Mathematika **46** (1999), 15–27.
[3] R. J. GARDNER, *Intersection bodies and the Busemann-Petty problem*, Trans. Amer. Math. Soc. **342** (1994), 435–445.
[4] R. J. GARDNER, *A positive answer to the Busemann-Petty problem in three dimensions*, Ann. of Math. **140** (1994), 435–447.
[5] R. J. GARDNER, *Geometric Tomography*, 2nd edition, Cambridge Univ. Press, Cambridge, 2006.
[6] R. J. GARDNER, A. KOLDOBSKY AND T. SCHLUMPRECHT, *An analytic solution to the Busemann-Petty problem on sections of convex bodies*, Ann. of Math. **149** (1999), 691–703.
[7] P. GOODEY AND W. WEIL, *Intersection bodies and ellipsoids*, Mathematika **42** (1995), 295–304.
[8] E. GRINBERG AND G. Y. ZHANG, *Convolutions, transforms, and convex bodies*, Proc. London Math. Soc. **78** (1999), 77–115.
[9] C. HABERL, *L_p intersection bodies*, Adv. Math. **217** (2008), 2599–2624.
[10] C. HABERL AND M. LUDWIG, *A characterization of L_p intersection bodies*, International Mathematics Research Notices, Art. ID 10548 (2006).
[11] N. J. KALTON AND A. KOLDOBSKY, *Intersection bodies and L_p spaces*, Adv. Math. **196** (2005), 257–275.
[12] A. KOLDOBSKY, *Intersection bodies and the Busemann-Petty problem*, C. R. Acad. Sci. Paris Ser. I Math. **325** (1997), 1181–1186.
[13] A. KOLDOBSKY, *Intersection bodies in $\mathbb{R}^4$*, Adv. Math. **136** (1998), 1–14.
[14] A. KOLDOBSKY, *Intersection bodies, positive definite distributions, and the Busemann-Petty problem*, Amer. J. Math. **120** (1998), 827–840.
[15] A. KOLDOBSKY, *Second derivative test for intersection bodies*, Adv. Math. **136** (1998), 15–25.
[16] A. KOLDOBSKY, *A functional analytic approach to intersection bodies*, Geom. Funct. Anal. **10** (2000), 1507–1526.
[17] A. KOLDOBSKY, *Fourier Analysis in Convex Geometry*, American Mathematical Society Press, Providence, 2005.
[18] A. KOLDOBSKY, G. PAOURIS AND M. ZYMONOPOULOU, *Complex Intersection Bodies*, J. Lond. Math. Soc. **88** (2013), 538–562.
[19] F. H. LU, W. H. MAO AND G. S. LENG, *On star duality of mixed intersection bodies*, J. Inequal. Appl., Article ID 39345, 12 pages (2007).
[20] M. LUDWIG, *Intersection bodies and valuations*, Amer. J. Math. **128** (2006), 1409–1428.

[21] E. LUTWAK, *Intersection bodies and dual mixed volumes*, Adv. Math. **71** (1988), 232–261.
[22] E. LUTWAK, *Centroid bodies and dual mixed volumes*, Proc. London Math. Soc. **60** (1990), 365–391.
[23] L. A. SANTALÓ, *Integral Geometry and Geometric Probability*, Addison-Wesley, Reading, MA, 1976.
[24] R. SCHNEIDER, *Convex Bodies: The Brunn-Minkowski Theory*, second ed., Cambridge University Press, Cambridge, 2014.
[25] W. WANG, R. G. HE AND J. YUAN, *On mixed complex intersection bodies*, Math. Inequal. Appl. **2** (2015), 419–528.
[26] W. D. WANG AND Y. N. LI, *Busemann-Petty problem for general L_p-intersection bodies*, Acta Mathematica Sinica **31** (2015), 777–786.
[27] J. YUAN AND W. S. CHEUNG, *L_p-intersection bodies*, J. Math. Anal. Appl. **339** (2008), 1431–1439.
[28] C. J. ZHAO AND G. S. LENG, *Brunn-Minlowski inequality for mixed intersection bodies*, J. Math. Anal. Appl. **301** (2005), 115–123.
[29] G. Y. ZHANG, *A positive solution to the Busemann-Petty problem in $\mathbb{R}^4$*, Ann. of Math. **149** (1999), 535–543.