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THE COMPLEX L_p LOOMIS–WHITNEY INEQUALITY

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Abstract. The complex L_p Loomis-Whitney inequality for complex isotropic measures is established, which extends the real version of the L_p Loomis-Whitney inequality for isotropic measures due to the first two authors.

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Paper MIA-21-27

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