JOINT WEAK TYPE INTERPOLATION
ON LORENTZ–KARAMATA SPACES

MICHAL BATHORY

Abstract. We present sharp interpolation theorems, including all limiting cases, for a class of quasilinear operators of joint weak type acting between Lorentz-Karamata spaces over σ-finite measure. This class contains many of the important integral operators. The optimality in the scale of Lorentz-Karamata spaces is also discussed. The proofs of our results rely on a characterization of Hardy-type inequalities restricted to monotone functions and with power-slowly varying weights. Some of the limiting cases of these inequalities have not been considered in the literature so far.

Mathematics subject classification (2010): 26D10, 46E30, 46B70, 47B38, 47G10.
Keywords and phrases: Real interpolation, joint weak type operators, Lorentz-Karamata spaces, Hardy inequalities.

REFERENCES

[1] C. BENNETT AND K. RUDNICK, *On Lorentz-Zygmund spaces*, Dissertationes Math. (Rozprawy Mat.), **175** (1980), 1–67.

[2] C. BENNETT AND R. SHARPLEY, *Interpolation of Operators*, Academic Press, Inc., Boston, 1988.

[3] N. H. BINGHAM, C. M. GOLDIE, AND J. L. TEUGELS, *Regular Variation*, Cambridge University Press, Cambridge, 1987.

[4] M. CARRO AND J. SORIA, *Weighted Lorentz spaces and the Hardy operator*, J. Funct. Anal. **112**, 2 (1993), 480–494.

[5] D. E. EDMUNDS, P. GURKA, AND B. OPIC, *Double exponential integrability of convolution operators in generalised Lorentz-Zygmund spaces*, Indiana Univ. Math. J. **44**, 1 (1995), 19–43.

[6] D. E. EDMUNDS, R. KERMAN AND L. PICK, *Optimal Sobolev imbeddings involving rearrangement-invariant quasinorms*, J. Funct. Anal. **170**, 2 (2000), 307–355

[7] W. D. EVANS, B. OPIC AND L. PICK, *Interpolation of operators on scales of generalized Lorentz-Zygmund spaces*, Math. Nachr. **182** (1996), 127–181.

[8] W. D. EVANS, B. OPIC AND L. PICK, *Real interpolation with logarithmic functors*, J. Inequal. Appl. **7**, 2 (2002), 187–269

[9] A. GOGATISHVILI, J. S. NEVES AND B. OPIC, *Optimality of embeddings of Bessel-potential-type spaces into Lorentz-Karamata spaces*, Proc. R. Soc. Edinb. **134A** (2004), 1127–1147.

[10] A. GOGATISHVILI, B. OPIC AND W. TREBELS, *Limiting reiteration for real interpolation with slowly varying functions*, Math. Nachr. **278**, 1–2 (2005), 86–107.

[11] A. GOGATISHVILI AND V. D. STEPANOV, *Reduction theorems for operators on the cones of monotone functions*, J. Math. Anal. Appl. **405**, 1 (2013), 156–172.

[12] P. GURKA AND B. OPIC, *Sharp embeddings of Besov-type spaces*, J. Comput. Appl. Math. **208**, 1 (2007), 235–269.

[13] L. V. KANTOROVICH AND G. P. AKILOV, *Functional Analysis*, 2nd ed., Pergamon Press, Oxford-Elmsford, N.Y., 1982.

[14] J. S. NEVES, *Lorentz-Karamata spaces, Bessel and Riesz potentials and embeddings*, Dissertationes Math. (Rozprawy Mat.) **405** (2002), 1–46.

[15] B. OPIC AND A. KUFNER, *Hardy-type inequalities*, Pitman Research Notes in Math. Series 219, Longman Sci & Tech., Harlow, 1990.

[16] E. SAWYER, *Boundedness of classical operators on classical Lorentz spaces*, Studia Math. **96**, 2 (1989), 145–158.

[17] G. SINNAMON, *Hardy's inequality and monotonicity*, Function Spaces, Differential Operators and Nonlinear Analysis (editors P. Drábek and J. Rákosník), Conference Proceedings, Milovy, Czech Republic, 2004.

[18] V. D. STEPANOV, *Weighted norm inequalities of Hardy type for a class of integral operators*, J. London Math. Soc. (2), **50**, 1 (1994), 105–120.

[19] V. D. STEPANOV, *The weighted Hardy's inequality for nonincreasing functions*, Trans. Amer. Math. Soc., **338**, 1 (1993), 173–186.

[20] A. ZYGMUND, *Trigonometric Series*, 3rd ed., vol. 1, Cambridge University Press, 2002.