# SOBOLEV'S INEQUALITIES FOR
# HERZ–MORREY–ORLICZ SPACES ON THE HALF SPACE

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*Abstract.* We introduce Herz-Morrey-Orlicz spaces on the half space, and study the boundedness of the Hardy-Littlewood maximal operator. As an application, we establish Sobolev's inequality for Riesz potentials of functions in such spaces, which is one of mixed norm type inequalities.

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