# ON CSISZÁR AND TSALLIS TYPE $f$–DIVERGENCES
# INDUCED BY SUPERQUADRATIC AND CONVEX FUNCTIONS

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*Abstract.* In this paper, Csiszár and Tsallis $f$-divergences are studied for superquadratic and convex functions. Some comparison theorems for two divergences are provided. The obtained results, when used for nonnegative superquadratic functions, give some refinements of the original inequalities corresponding to nonnegative convex functions. Some majorization assumptions for the involved matrix are simplified from column stochasticity to entrywise-nonnegativity.

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