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ON THE TRIANGLE EQUALITY

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Abstract. Given a norm on a vector space a natural question is when for given vectors the triangle equality holds, i.e., when the norm of a sum of vectors is equal to the sum of norms of vectors. It is well known that if the space is strictly convex, then the triangle equality holds if and only if the vectors belong to the same half-line emanating from the origin. In the note we give necessary and sufficient geometric conditions for the triangle equality in an arbitrary normed vector space. The conditions can be checked once the unit sphere is known.

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