

POMPEIU–LIKE THEOREMS FOR THE MEDIANS OF A SIMPLEX

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Abstract. A theorem of Pompeiu states that the distances from the vertices of an equilateral triangle to an arbitrary point in its plane can serve as the side lengths of a triangle. A similar theorem holds for the medians of any triangle. Generalizations to higher dimensions of Pompeiu’s theorem are established by M. Fiedler in 1977 and by Gh. Al-Afifi et al in a forthcoming paper. This paper is concerned with similar possible higher dimensional theorems for the medians.

Mathematics subject classification (2010): 51M04, 51M16, 51M15, 51M25, 51M20.

Keywords and phrases: Content, facet, generalized medians, simplex, isodynamic simplex, medians, Pompeiu’s theorem.

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