Mathematical
Inequalities
& Applications
Volume 21, Number 2 (2018), 575–584
doi:10.7153/mia-2018-21-42

# SOME PROPERTIES OF ZIPF–MANDELBROT LAW AND HURWITZ $\zeta$–FUNCTION

JULIJE JAKŠETIĆ, ĐILDA PEČARIĆ AND JOSIP PEČARIĆ

*Abstract.* In this paper we deal with analytical properties of the Zipf-Mandelbrot law. If total mass of this law is spread all over positive integers we come to Hurwitz $\zeta$-function. As we show, it is very natural first to examine properties of Hurwitz $\zeta$-function to derive properties of Zipf-Mandelbrot law. Using some well-known inequalities such as Chebyshev's and Lyapunov's inequality we are able to deduce a whole variety of theoretical characterizations that include, among others, log-convexity, log-subadditivity, exponential convexity.

*Mathematics subject classification* (2010): Primary 26D15, Secondary 26D20.
*Keywords and phrases*: Zipf-Mandelbrot law, Hurwitz $\zeta$-function, log-convexity, Chebyshev's inequality, Lyapunov's inequality.

## REFERENCES

[1] G. BASTIEN, M. ROGALSKI, *Convexité, complète monotonie et inégalités sur les fonctions zêta et gamma, sur les fonctions de opérateurs de Baskakov et sur les fonctions arithmétiques*, Can. J. Math. **54**, 5 (2002), 916–944.
[2] S. FRONTIER, *Diversity and structure in aquatic ecosystems*, Mar. Biol. Ann. Rev. **23** (1985), 253–312.
[3] L. HORVÁTH, Đ. PEČARIĆ, J. PEČARIĆ, *Estimations of $f$ and Rényi divergences by using a cyclic refinement of the Jensen's inequality*, Bull. Malays. Math. Sci. Soc. (accepted).
[4] J. JAKŠETIĆ, J. PEČARIĆ, *Exponential Convexity method*, J. Convex Anal. **20**, 1 (2013), 181–197.
[5] A. LELU, *Jean-Baptiste Estoup and the origins of Zipf's law: a stenographer with a scientific mind (1868–1950)*, Boletiín de Estadiística e Invenstigación Operativa 6–77, **30**, 1 (2014), 66–77.
[6] B. MANDELBROT, *An information theory of the statistical structure of language*, In Jackson, W., editor, Communication Theory, New York, Academic Press, 1953.
[7] D. Y. MANIN, *Mandelbrot's Model for Zipf's Law: Can Mandelbrot's Model Explain Zipf's Law for Language?*, **16**, 3 (2009), 274–285.
[8] D. MOUILLOT, A. LEPRETRE, *Introduction of relative abundance distribution (RAD) indices, estimated from the rank-frequency diagrams (RFD), to assess changes in community diversity*, Environmental Monitoring and Assessment **63**, 2 (2000), 279–295.
[9] J. PEČARIĆ, *Nejednakosti*, Hrvatsko matematičko društvo, Zagreb, 1996.
[10] J. PEČARIĆ, F. PROSHAN, Y. L. TONG, *Convex functions partial ordering and statistical applications mathematics in Science and Engineering*, vol. 187., Academic Press, 1992.
[11] J. B. WILSON, *Methods for fitting dominance/diversity curves*, **2** (1991), 35–46.
[12] G. K. ZIPF, *Human Behavior and the Principle of Least Effort*, Addison-Wesley, Reading, MA, 1949.