

INEQUALITIES AMONG EIGENVALUES OF DIFFERENT SELF-ADJOINT DISCRETE STURM-LIOUVILLE PROBLEMS

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Abstract. In this paper, inequalities among eigenvalues of different self-adjoint discrete Sturm-Liouville problems are established. For a fixed discrete Sturm-Liouville equation, inequalities among eigenvalues for different boundary conditions are given. For a fixed boundary condition, inequalities between the n -th eigenvalues for two different equations are given. These results are obtained by applying continuity and discontinuity of the n -th eigenvalue function, monotonicity in some direction of the n -th eigenvalue function, which were given in our previous papers, and natural loops in the space of boundary conditions. Some results generalize the relevant existing results about inequalities among eigenvalues of different Sturm-Liouville problems.

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