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QUADRATIC INTERPOLATION OF THE HEINZ MEANS

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Abstract. The main goal of this article is to present several quadratic refinements and reverses of the well known Heinz inequality, for numbers and matrices, where the refining term is a quadratic function in the mean parameters. The proposed idea introduces a new approach to these inequalities, where polynomial interpolation of the Heinz function plays a major role. As a consequence, we obtain a new proof of the celebrated Heron-Heinz inequality proved by Bhatia, then we study an optimization problem to find the best possible refinement. As applications, we present matrix versions including unitarily invariant norms, trace and determinant versions.

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