TOPICAL FUNCTIONS: HERMITE–HADAMARD TYPE INEQUALITIES AND KANTOROVICH DUALITY

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Abstract. For a certain class of elementary functions consisting of min-type functions, we apply techniques from abstract convex analysis to study Hermite-Hadamard type inequalities for increasing and plus-homogeneous (topical) functions. Some examples of such inequalities for functions with the special domains are given as well. In the next part, we study Kantorovich duality for the optimal mass transportation problems whenever the cost function is a min-type function. In this case, some pricing criteria are established as well.

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