

FUNCTIONS WITH STRICTLY CONVEX EPIGRAPH

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Abstract. The aim of this paper is to provide a full and simple characterization of functions with domain in an *arbitrary* topological real vector space whose epigraph is *strictly* convex. In order to achieve this description, we first give both affine and topological properties of strictly convex subsets of general topological real vector spaces. Then, we investigate the relationship between functions and their epigraph from the geometric and topological viewpoints.

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