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TRACE AND EXTENSION THEOREMS RELATING BESOV SPACES TO WEIGHTED AVERAGED SOBOLEV SPACES

ARIEL BARTON

Abstract. There are known trace and extension theorems relating functions in a weighted Sobolev space in a domain Ω to functions in a Besov space on the boundary $\partial\Omega$. We extend these theorems to the case where the Sobolev exponent p is less than one by modifying our Sobolev spaces to consider averages of functions in Whitney balls. Averaged Sobolev spaces are also of interest in the applications in the case where $p > 1$, and so we also provide trace and extension results in that case. Finally, we provide some comparable results for Neumann traces and extensions.

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