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ON THE BANACH–MAZUR DISTANCE BETWEEN THE CUBE AND THE CROSSPOLYTOPE

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Abstract. In this note we study the Banach-Mazur distance between the n-dimensional cube and the crosspolytope. Previous work shows that the distance has order $\sqrt{n}$, and here we will prove some explicit bounds improving on former results. Even in dimension 3 the exact distance is not known, and based on computational results it is conjectured to be $\frac{9}{5}$. Here we will also present computer based potentially optimal results in dimension 4 to 8.

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