ON THE PTOLEMY CONSTANT OF SOME CONCRETE BANACH SPACES

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Abstract. In this paper, we firstly consider the relations involving the Ptolemy constant of the norms $\|.\|_\psi$ and $\|.\|_\phi$, where the convex functions and ψ and ϕ are comparable. Secondly, we determine this constant when norm is a mean of two norms. Finally, the constant was calculated for some concrete Banach spaces.

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