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NEW ASYMPTOTIC EXPANSION AND ERROR BOUND FOR STIRLING FORMULA OF RECIPROCAL GAMMA FUNCTION

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Abstract. Studying the problem about if certain probability measures are determinate by its moments [4, 8, 10] is useful to know the asymptotic behavior of the probability densities for large values of argument. This requires, previously, the knowledge of the asymptotic expansion of reciprocal Gamma function $1/\Gamma(z)$ when $\Re z$ is large and $\Im z$ is fixed [8]. Then, the well known Stirling formula for large $|z|$ of the Gamma function $\Gamma(z)$ or its reciprocal $1/\Gamma(z)$ is not appropriate for this problem. So, the main aim of this paper is to obtain a new asymptotic expansion for reciprocal Gamma function valid for large $\Re z$ and establish a new explicit error bound for the first term of this expansion, that is, the Stirling formula.

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