

NEW CARLSON–BELLMAN AND HARDY–LITTLEWOOD DYNAMIC INEQUALITIES

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Abstract. In this paper, we will prove some new dynamic inequalities of Carlson and Hardy–Littlewood types on an arbitrary time scale \mathbb{T} . These inequalities as special cases contain the classical continuous and discrete Carlson–Bellman and Hardy–Littlewood type inequalities. The results will be proved by employing the time scales Hölder inequality, some algebraic inequalities and some basic lemmas designed and proved for this purpose.

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